

Pseudo rapidity

$$\eta = -\ln(\tan(\frac{\theta}{2})) \quad (1)$$

$$\cos(\theta) = 2 \cos^2(\frac{\theta}{2}) - 1 \quad (2)$$

$$\sin(\theta) = 2 \cos(\frac{\theta}{2}) \sin(\frac{\theta}{2}) \quad (3)$$

$$\tan(\frac{\theta}{2}) = \frac{\sin(\frac{\theta}{2})}{\cos(\frac{\theta}{2})} = \frac{2 \cos(\frac{\theta}{2}) \sin(\frac{\theta}{2})}{2 \cos^2(\frac{\theta}{2})} \quad (4)$$

$$= \frac{\sin(\theta)}{1 + \cos(\theta)} \quad (5)$$

$$= \left(\frac{\sin^2(\theta)}{\{1 + \cos(\theta)\}^2} \right)^{\frac{1}{2}} \quad (6)$$

$$= \left(\frac{1 - \cos^2(\theta)}{\{1 + \cos(\theta)\}^2} \right)^{\frac{1}{2}} \quad (7)$$

$$= \left(\frac{1 - \cos(\theta)}{1 + \cos(\theta)} \right)^{\frac{1}{2}} \quad (8)$$

$$= \left(\frac{p - p_z}{p + p_z} \right)^{\frac{1}{2}} \quad (9)$$

Hence,

$$\eta = \frac{1}{2} \ln \left(\frac{p + p_z}{p - p_z} \right) \quad (10)$$

The rapidity is defined as

$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) \quad (11)$$

By the Lorentz boost along the z -direction, it is shifted without changing the form. Therefore, if E is large and $E \sim p$, the same is true for the pseudo rapidity, η .