Brems energy sampling

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For treating electron bremshtralung, we need to get the total brems cross-section (say, probability or brems occurence / r.l) and must be able to sample the emitted gamma ray energy. Normally, the total cross-section diverges when the minimum gamma ray energy is set to 0 (though, if the LPM is effective, divergnece dose not happen).

The gamma ray energy is usually expressed by $k = E_\gamma/E_e$ where E_e is the electron totalc energy. To avoid divergence, we normally set a minimum value of $k = k_m$ above which gamma ray energy is sampled. This is applied also for the LPM case.

However, some times we may want to use a lower k_m than predefined one. To prepare a sampling table for each different k_m is not practical.

1 Sampling Method

We express the differential brems cross-section by

$$\frac{d\sigma}{dk} = f(k) \tag{1}$$

This is dependent on E_e. For a typical k_m, we make a table with various E_e's. First, the total cross-section, i.e, integration from $k = k_m$ to $1 - m_e/Ee$,

$$T(k_m) = \int_{k_m} f(k)dk \tag{2}$$

is tabulated.

For $k \sim 1$, $f(k)$ is not a simple function, so we must also make a sampling table for k, beforehand. For a lower k_c ($< k_m$,), $T(k_c)$ can be obtained by a simple formula, since $f(k)$ becomes a simple function for sufficiently small k. That is, when the LPM is to be applied:

$$f(k) = f_1 \equiv f_m \cdot (k/k_m)^{-1/2} = f_{m1}/\sqrt{k} \tag{3}$$

where $f_m = f(k_m)$ and $f_{m1} = f_m\sqrt{k_m}$. When the LPM should not be applied:

$$f(k) = f_2 \equiv f_m \cdot (k/k_m)^{-1} = f_{m2}/k \tag{4}$$

where $f_{m2} = f_m k_m$. Both of f_{m1} and f_{m2} are tabulated together with $T(k_m)$

$$T(k_c) = \int_{k_c} f(k)dk = \int_{k_c}^{k_m} f(k)dk + T(k_m) \tag{5}$$

Defining

$$t_i(k_c) \equiv \int_{k_c}^{k_m} f_i dk = \left\{ \begin{array}{ll} 2f_{m1}(\sqrt{k_m} - \sqrt{k_c}) & (i=1) \\ f_{m2}\log(k_m/k_c) & (i=2) \end{array} \right. \tag{6}$$

We can express

$$T(k_c) = t_i(k_c) + T(k_m) \tag{7}$$

For a given uniform random number $0 < u < 1$, if $u > t_i(k_c)/T(k_c)$, we may sample k by using the sampling table. Otherwize, we may sample k from f_i. Using a new u,

$$k = \left\{ \begin{array}{ll} k_c(1 + u(\sqrt{k_m/k_c} - 1))^2 & (i=1) \\ k_c(\frac{k_m}{k_c})^u & (i=2) \end{array} \right. \tag{8}$$

2 Summary

- Besides $T(k_m)$, f_{m1} and f_{m2} must be tabulated as a function of E_e.
- If f_{m1} or f_{m2} is not given in the table, old style format is assumed, and giving an new k_c is not allowed (negelcted)
- If a new k_c is larger than the current k_m, it is neglected (or $k_c = k_m$ is forced.)
- Normally the minimum gamma ray energy must be ≥ 1 keV. (It can be smaller without error; If E_e <100 MeV, the absolute E_γ minimuum has been set to be 1 keV in default). At higher energies, $k_m = 10^{-5}$ is default and this is enough small for many of applications. If E_e is as high as 10 TeV, it means the minimum gamma energy becomes 100 MeV. One may think this is too high, but for many applications where energy deposit is measured, results with this cut would give a result which is almost the same one by the default setup
- The total cross-section table, say, BrTXL(mxBrTXL), is now made to be BrTXL(2, mxBrTXL). The first index is for the total cross-section, and the second one is for f_{m1} or f_{m2}.

  BrTXL, BrTXH, BrTXS, BrTXS2 are also changed.