The LPM and geomagnetic effects on the development of air showers in the GZK cutoff region

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1 Introduction

The Landau-Pomeranchuk-Migdal (=LPM) effect[1] is a well known process which reduces the cross-sections of bremsstrahlung and pair creation at high energies and in a dense medium. However, experimental evidence is rather meager, especially at very high energies. Experimental examination of the effect in the 100 TeV region in lead was tried in ref.[2]; the showers presumably induced by an electron or photon showed a clear difference in cascade shower development from that assuming no LPM effect. Then, a similar effect is expected to occur for air showers initiated by photons of energy $> 10^{19}$ eV, since the cross-section scales (except for some numerical factors) with the product of energy and medium density.

Although a variety of magnetic effects might occur for high energy electrons and photons in a strong field, only magnetic bremsstrahlung (synchrotron radiation) and pair creation are important in the earth environment at the Greisen-Zatsepin-Kuzmin (GZK) cutoff energies[4]. The possibility of having photon primaries in the GZK cutoff region is unknown but one could make plausible models of photon primaries which generate air showers with muons comparable to the proton primary case (For example, see ref.[3] and references therein). There are extremely high energy events that came from almost the same directions in rather a short time interval[5], which may suggest neutral primaries. Therefore it is worth while to investigate what will happen when extremely high energy photons come to the Earth.

Air shower development with the LPM and/or the geomagnetic effect have been studied in earlier works[3],[9]. In this talk, we examine the same problem using a Monte Carlo code, Cosmos[6], and address the following questions.

- The LPM effect on the proton primary showers.
- The LPM and magnetic effects on the photon primary showers.
  - The north-south asymmetry of apparent shower size due to the magnetic field line configuration.

2 Cross-sections

When magnetic field strength, B, becomes so strong that electron's cyclotron frequency equals to its mass, m, i.e., $eB/m \sim m$, a quantum mechanical effect is expected to appear. The critical field is thus, $B_c = m^2/e = 4.4 \times 10^{13}$ Gauss. This is quite a large value as compared with the geomagnetic field. However, there is also a relativistic effect, and a relevant parameter for the quantum effect is [7],

$$\Upsilon = \frac{B}{B_c}\frac{E}{m} \tag{1}$$

where E is the electron energy. If $\Upsilon \sim 1$, i.e., $E \sim 5 \times 10^{19}$ eV at the earth environment, the quantum effect becomes to play an important role.

For a photon, we may divide the photon energy (E_γ) by 2, since the energy goes into two electrons at pair creation and the relevant parameter is

$$\chi = \frac{1}{2}\frac{B}{B_c}\frac{E_\gamma}{m} \tag{2}$$

Appropriate formulas for magnetic bremsstrahlung and pair creation are given in an excellent summary article[8] . We show formulas for a particle moving with the right angle to the magnetic field, $\vec{B}$. The Lorentz boost tells us that we may use $B\sin\theta$ instead of B if the angle between the momentum and $\vec{B}$ is θ.

2.1 Magnetic bremsstrahlung

The bremsstrahlung cross-section, expressed in terms of the probability of emitting a photon of fractional energy, $v = E_\gamma/E$ and $v + dv$, per unit length is

$$\varphi(E.v.\Upsilon)dv = \frac{\sqrt{3}}{2\pi}\frac{S_c}{E}\Upsilon\varphi^*(v.\Upsilon)dv \qquad \varphi^*(v.\Upsilon) = \frac{1-v}{v}\kappa_1(2\xi) + v\kappa_2(2\xi) \tag{3}$$

where $S_c = \alpha m/2\pi\lambda_c$ is the synchrotron conversion rate having a numerical value of 9.66 TeV/cm (α is the fine structure constant and λ_c the electron compton wave length). ξ is defined as

$$\xi = \frac{1}{3\Upsilon}\frac{v}{1-v} \tag{4}$$

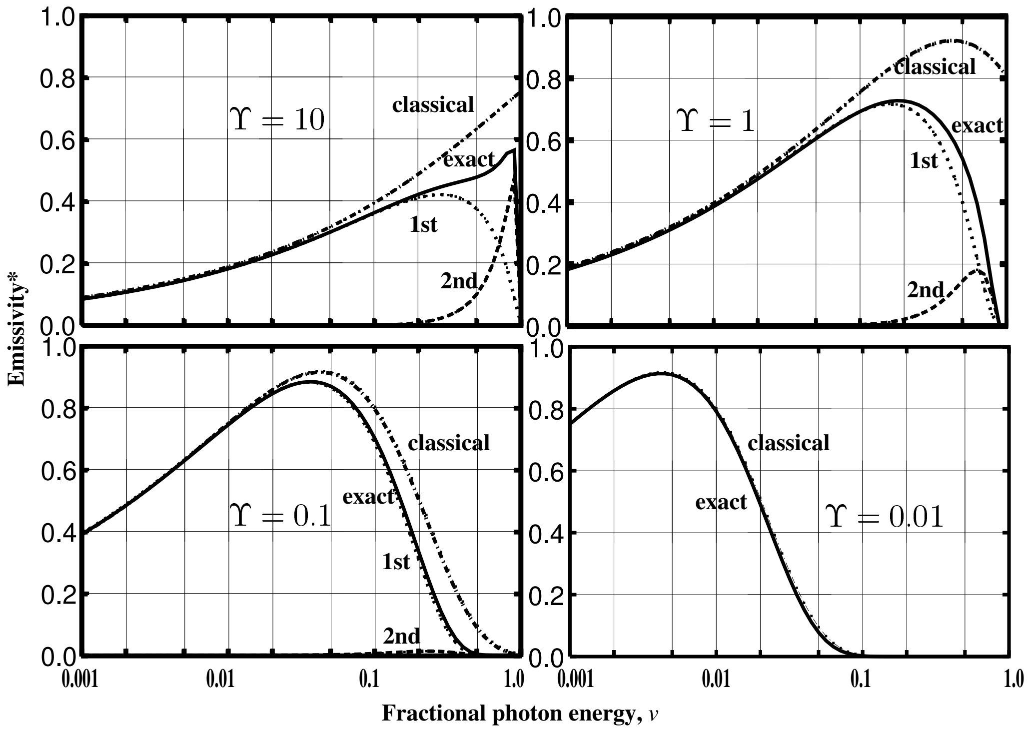

Figure 1: Magnetic bremsstrahlung spectra are show in the form of emissivity ($v\varphi^*(v)$) for $\Upsilon = 10, 1, 0.1$ and 0.01. The first and second terms in $\varphi^*(v)$ are separately given.

The two functions in φ^* are

$$\kappa_1(z) = z\int_z^\infty K_{5/3}(x)dx \quad (5)$$

$$\kappa_2(z) = zK_{2/3}(z) \quad (6)$$

where K is the modified Bessel function of a fractional order. The asymptotic forms of these two are

$$\kappa_1 \rightarrow \begin{cases} 2z^{\frac{1}{3}} & \text{for } z \rightarrow 0 \\ 1.3\sqrt{z}\cdot e^{-z} & \text{for } z \rightarrow \infty \end{cases} \qquad \kappa_2 \rightarrow \begin{cases} \kappa_1/2 & \text{for } z \rightarrow 0 \\ \kappa_1 & \text{for } z \rightarrow \infty \end{cases} \quad (7)$$

The second term in φ^* contributes only if $\Upsilon \gg 0.1$ at $v \sim 1$.
The classical synchrotron radiation formula is resumed by dropping the second term in φ^* and replacing $(1-v)$ by 1 both in Eqs.3 and 4. In Fig.1, bremsstrahlung emissivity, $v\varphi^*(v)$, is shown for several Υ values.

The energy loss rate by bremsstrahlung is,

$$-\frac{dE}{dt} = E\int_0^1 v\varphi(v)dv \propto \begin{cases} \Upsilon^2(1-6\Upsilon) & \Upsilon \rightarrow 0 \\ \Upsilon^{\frac{2}{3}} & \Upsilon \rightarrow \infty \end{cases} \quad (8)$$

For a fixed B, the mean free path for emitting one photon of arbitrary energy is

$$\frac{1}{\int \varphi(v)dv} \propto \begin{cases} const & \text{for } E \rightarrow 0 \\ E^2 & \text{for } E \rightarrow \infty \end{cases} \quad (9)$$

This means the cross-section itself decreases as energy goes higher in a fixed B field. However, the attenuation length, Λ, has large values at low energies, decreases rapidly with energy, reaches a minimum at certain energy ($\Lambda_{min} \sim 40$ km at $E \sim 1.5\times 10^{20}$ eV for 0.3 Gauss), and slowly increases at higher energies as seen in Fig.2.

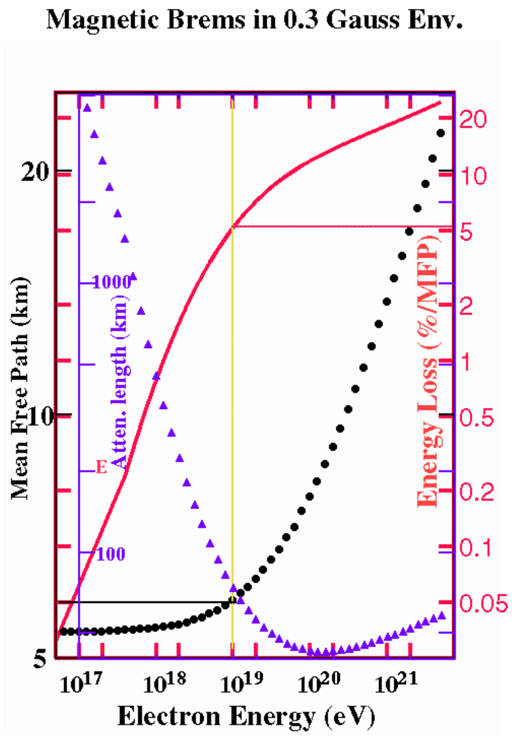

Figure 2: The magnetic bremsstrahlung mean free path for emitting one photon (circle), energy loss per m.f.p in percentage (solid line) and attenuation length (triangle) in a 0.3 Gauss environment.

2.2 Magnetic pair creation

The magnetic pair creation cross-section can be written in a very similar form as the bremsstrahlung case. However, we show the following one for practical computation.

$$\phi(E_\gamma,\chi,v)dv = \frac{1}{3\pi\sqrt{3}}\frac{S_c}{E_\gamma}\frac{9-x^2}{1-x^2}K_{2/3}(\zeta)dv \quad (10)$$

where v is the fractional energy of one of the pair electrons, $x = 1-2v$, and

$$\zeta = \frac{4}{3\chi}\frac{1}{(1-v)^2} \quad (11)$$

ϕ is the probability of pair creation per unit length. The asymptotic form of the mean free path is

$$M.F.P = \frac{1}{\int \phi(v)dv} \propto \begin{cases} \frac{1}{\chi\exp(-\frac{4}{3\chi})} & \text{for } \chi \rightarrow 0 \\ \frac{E_\gamma^{2/3}}{B^{1/3}} & \text{for } \chi \rightarrow \infty \end{cases} \quad (12)$$

A graphical presentation of ϕ is in Fig.3. We may note the following two: 1) the m.f.p exponentially diverges at small energies, and this brings a sharp threshold energy at $\sim 3\times10^{19}$ eV for pair creation in the earth environment. 2) for a fixed B, the m.f.p also becomes longer at very high energies (see Fig.4).

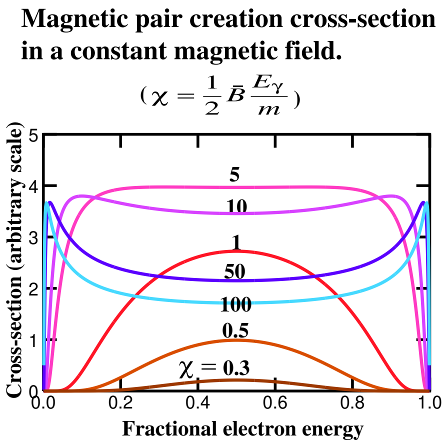

Figure 3: Magnetic pair creation cross-section in a constant B field

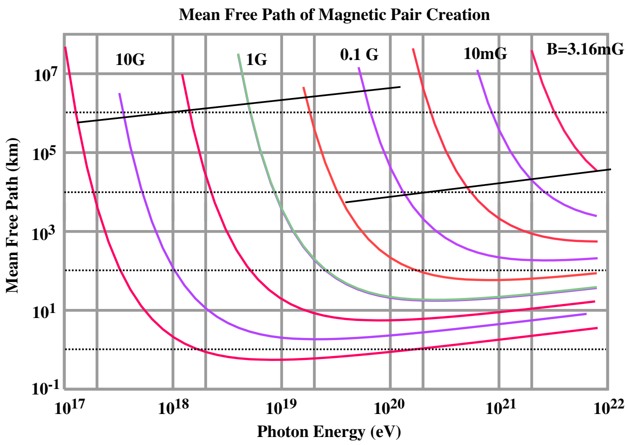

Figure 4: Magnetic pair creation mean free path as a function of photon energy in various B's.

2.3 The LPM effect

We use the cross-section for the LPM effect as described in Migdal's paper[1]. The formulas for bremsstrahlung and pair creation scale with the product of the energy (E) and air density (ρ). We show the probability of pair creation per radiation length and the fractional energy loss rate for bremsstrahlung in Fig.5 as a function of ρE. Both cross-sections drop as $1/\sqrt{\rho E}$ in high energy limit (Fig.5).

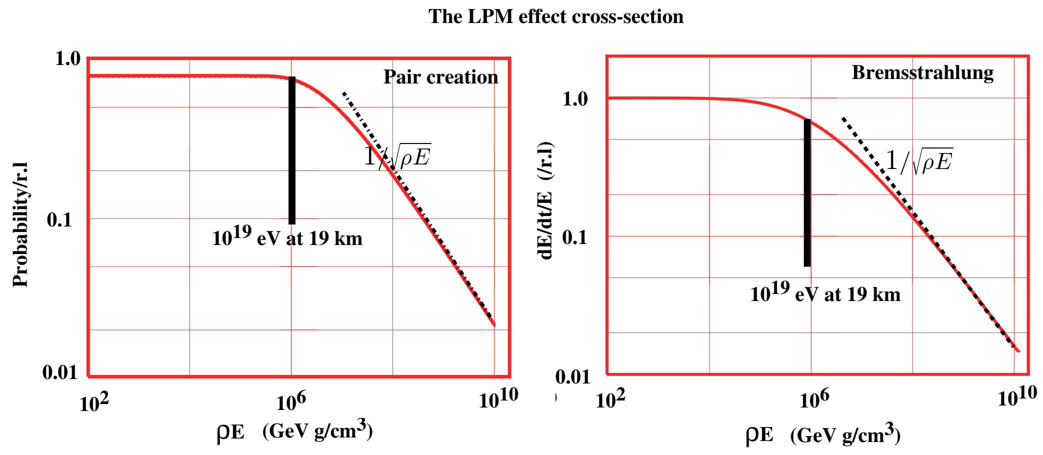

Figure 5: The LPM bremsstrahlung and pair creation cross-sections are shown as a function of the product of air density and energy

2.4 Break-even points

At very high altitudes where there is almost no matter, only magnetic effect works while at low altitudes matter bremsstrahlung and pair creation would dominate. Therefore there should be energy dependent break-even altitudes where the both effects equally compete. For a 0.3 Gauss environment, we get break-even points as shown in Fig.6

3 Results

We use the Cosmos code to obtain Monte Carlo results. Cosmos uses an accurate geomagnetic field configuration, using up to 8 terms of the Legendre expansion of the field.

3.1 The LPM effect on protonic showers

For proton primaries, the geomagnetic effect is completely negligible and the problem here is to investigate the LPM effect on air shower development. If the LPM effect is to work, we should have high energy photons ($> 10^{19}$ eV); the source of photons are normally π^0's but such high energy π^0's cannot decay before colliding with the air target. Another source is the η meson. The production ratio of η to π^0 is assumed to be ~20 % in Cosmos so that the LPM effect is expected to be not so large (non-decaying π^0's might have some effect on air shower development).

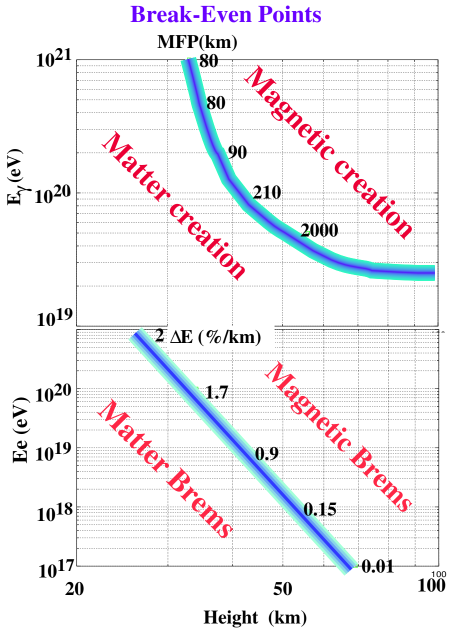

Figure 6: Break-even points where magnetic and matter pair creation (upper) and bremsstrahlung (lower) compete in a 0.3 Gauss field. Numbers attached to each curve are m.f.p for pair creation and percentage energy loss per km for bremsstrahlung.

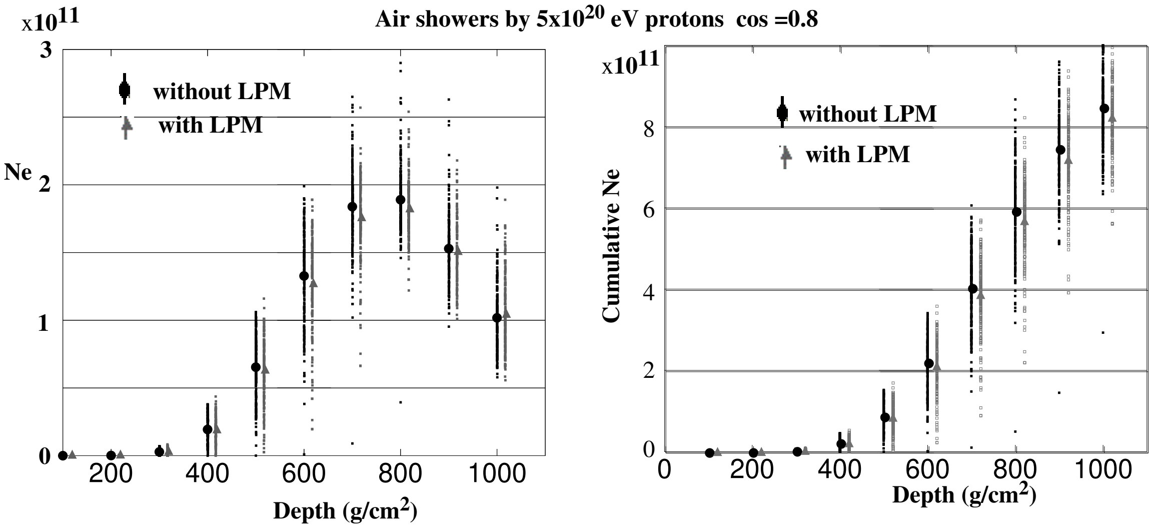

Figure 7: Transition of air shower size for 5×10^{20} eV proton primaries with and without the LPM effect. 200 showers are superimposed for each case. Left: Size, N_e is in the ordinary scale. Right: The cumulative size in ordinary scale.

In order to see the small effect clearly, we show in Fig.7 the transition of the air shower size in normal scale for 5×10^{20} eV primaries with and without the LPM effect. The effect is only 10 % level. In the figure is shown a cumulative size (we simply add the sizes at shallower depths); this will give a clue to estimate the effect on the total absorption type observation such as Cehrenkov light observation. It will also help Fly's eye type observation.

3.2 The LPM and geomagnetic effects on photon primary showers

3.2.1 A schematic picture

To have a quick idea of what happens when a very high energy photon enters into the earth environment, we show a schematic picture when a photon of 10^{20} eV comes vertically to the earth (at Yangbajin, Tibet); the energy spectra of electrons and photons at 100 km above sea level are shown in Fig.8 The first pair creation typically takes place at $\sim$ 1000 km high, and the resultant pair electrons radiate $\sim$260 photons of energy $> 10^{12}$ eV until they reach 100 km. The average photon energy is $\sim 3.6 \times 10^{17}$ eV. These photons cannot make pair creation because of a sharp threshold at $> 5 \times 10^{19}$ eV at very high altitudes. Therefore, at 100 km above sea level, we have essentially two electrons, of which the average energy is $\sim 2.7 \times 10^{18}$ eV. Almost all of these electrons and photons are contained within a radius of 1 mm circle.

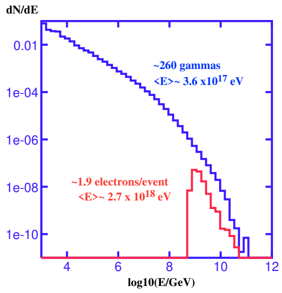

Figure 8: Average energy spectra of magnetic cascade at 100 km above sea level for a photon primary of 10^{20} eV.

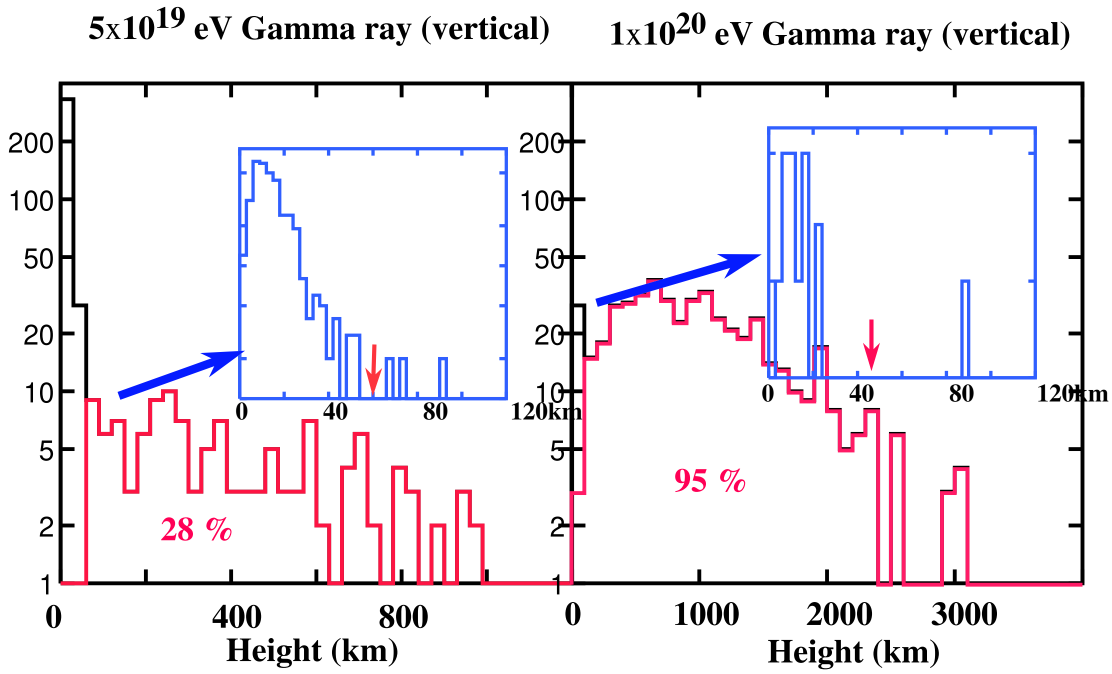

Figure 9: Distribution of the first pair creation height. Heights > 50 km are mostly due to magnetic pair creation. Left: For photon primary of 5×10^{19} eV. About 28 % makes magnetic pair creation. Right: For 1×10^{20} eV. $\sim$95 % makes magnetic pair creation.

We note that there is a sharp (lower-energy-side) cutoff at $\sim 6 \times 10^{17}$ eV in the electron energy spectrum (no electrons can go down below this energy; existence of such a cutoff can easily be verified by assuming a dipole field and classical energy loss rate proportional to E^2). The first pair creation height distribution is shown in Fig.9 for two photon energies.

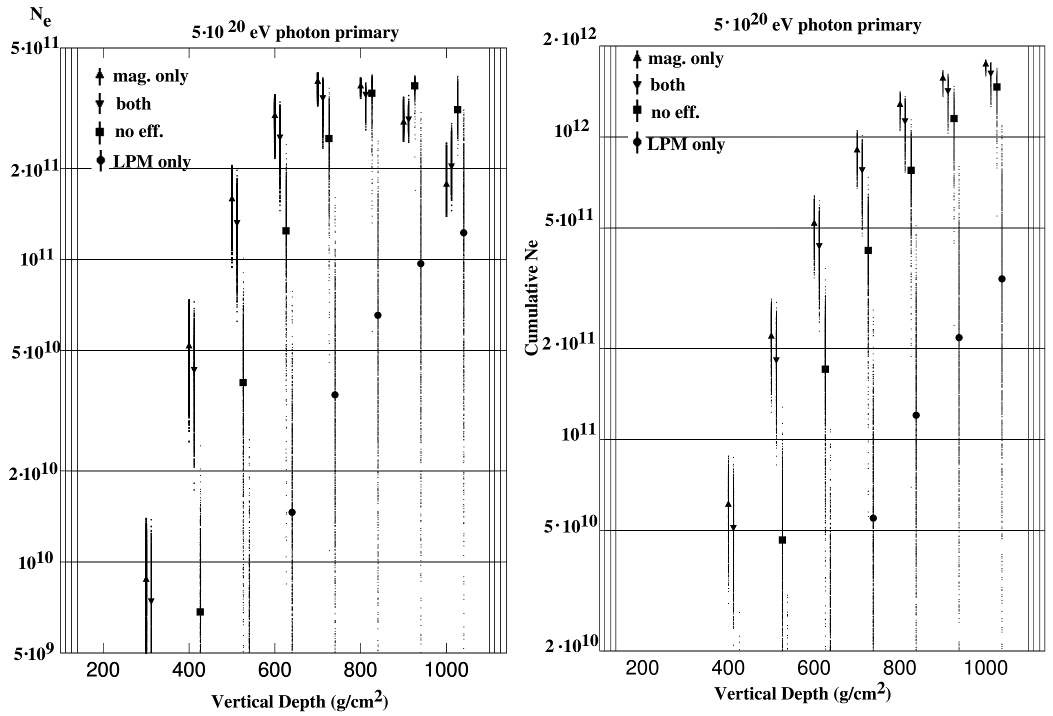

Figure 10: Transition of N_e (left) and cumulative N_e (right) for 5×10^{20} eV photon primaries. Cases with 4 different assumptions are plotted with a slight shift of depth. Each case includes 1000 showers. Triangle etc show the average values.

3.2.2 Transition curve

We compare air shower transitions in Fig.10 for showers: 1) with only the geomagnetic effect, 2) with the full (LPM+geomagnetic) effect, 3) without any special effect and 4) with only the LPM effect. The energy of the primary photons is 5×10^{20} eV. Since the magnetic effect is sensitive to the angle between the primary direction and the magnetic field line, we assumed uniform azimuthal angle distribution with a fixed zenith angle (0.8 in cosine) for the primary angular distribution.

We note the transitions are very much different depending on the assumption. Since the energy estimation primarily depends on the shower size, we should pay a special attention to how to get reliable primary energy if we assume the shower is initiated by a photon.

3.2.3 The north-south asymmetry

In what follows, we assume that both the LPM and geomagnetic effects are real. If a primary photon comes with an angle $\theta \sim 0$ (angle between the filed line and the photon momentum), the geomagnetic effect is almost absent, and the LPM effect delays the development of shower considerably. On the other hand, if θ is not close to 0, the shower develops quickly and fluctuation is very small. Therefore we may observe some kind of north-south asymmetry in observed shower sizes, since the geomagnetic field line is directed from the south to north with a considerable dip angle in may places.

As an example of the observation site, we take Yangbajin, Tibet where the longitude is $\sim 90°$, latitude $\sim 30°$ and the field dip angle is downwards with $\sim 45°$. We divide showers into two classes: One is with $B\sin\theta < 0.2$ (azimuthal angle $\varphi \approx 230 \sim 310°$ which is measured from the magnetic east counter-clock-wise) and the other with $B\sin\theta > 0.45$ ($\varphi \approx 30 \sim 150°$). There is a systematic difference between showers of these two classes for primary energy 5×10^{20} eV(Figs.11, 12). However, the north-south asymmetry is not critical at this energy, especially at near sea level.

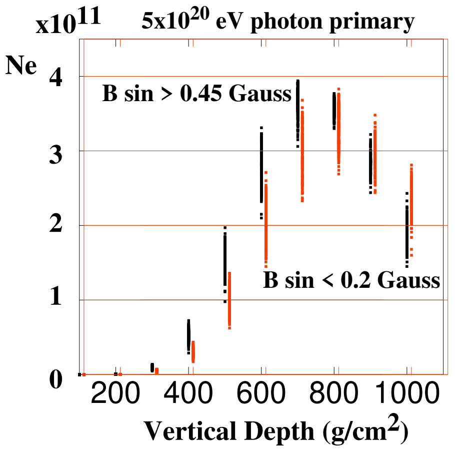

Figure 11: Difference between N_e for 5×10^{20} eV showers coming from the north ($B \sin\theta > 0.45$ Gauss) and those from the south ($B \sin\theta < 0.2$ Gauss). The zenith angle is 0.8 in cosine.

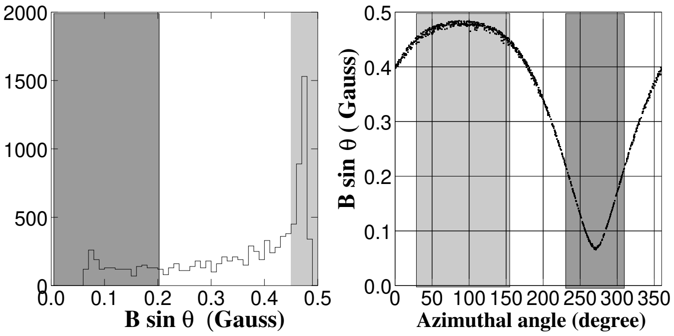

Figure 12: Left: $B \sin\theta$ distribution. The showers with large and small $B \sin\theta$ are extracted from the shadow regions. Right: Corresponding azimuthal angle region.

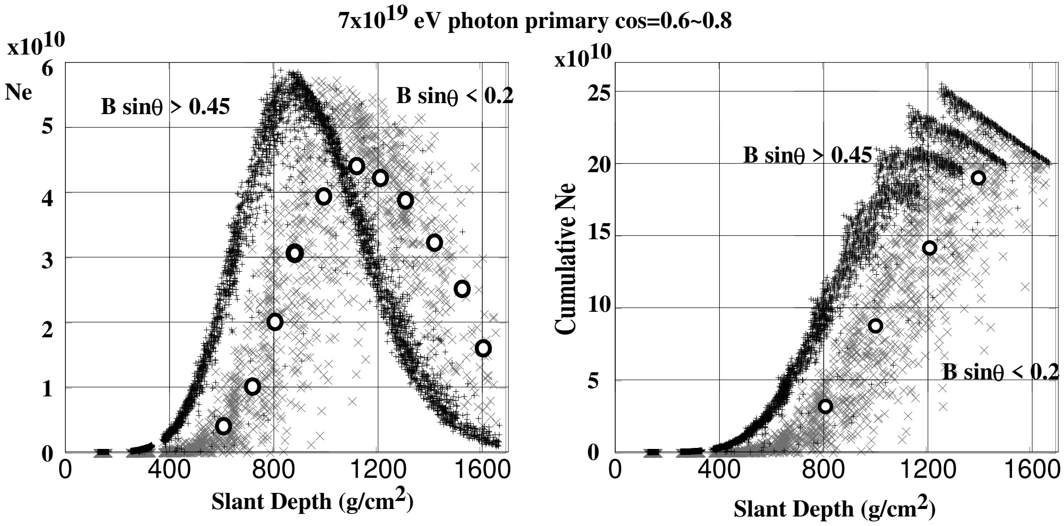

Figure 13: The north-south asymmetry for 7×10^{19} eV photon primary showers. Plus signs (+: thicker points) show showers with $B \sin\theta > 0.45$ Gauss They have very small fluctuation. Crosses (x: thinner points) are those with $B \sin\theta < 0.2$ and suffer the LPM effect to show large fluctuation. The primary zenith angle was sampled isotropically in cosine=0.6 $\sim$ 0.8. The circles show the average values for the showers with $B \sin\theta < 0.2$. Left: N_e, right: cumulative N_e. Note that the slant depth is used in this figure.

This is due to the fact that the magnetic effect almost always works at this high energy even if $B \sin\theta$ is small. If we go down to 7×10^{19} eV, the pair creation cross-section is reduced and many showers suffer the LPM effect, and hence the difference between the two classes becomes sizable. This is clearly shown in Fig.13. It is also important to note that, (especially, if one is using Akeno type observation, i.e., one layer sampling), the energy determination of a shower needs a careful treatment depending on the observation depth, as one may understand it from that figure. In other words, if we want to see the north-south asymmetry, we may go to a higher land.

We may also note that the north-south asymmetry should have a sharp threshold at around 3×10^{19} eV. It is unlikely that all primaries are photons at these energies so one should study how much percentage of photon primary is necessary if one is able to see the north-south asymmetry.

4 Conclusions

1. Proton primary showers.
   - The LPM effect is small even at 5×10^{20} eV. The effect in the shower size at sea level is only 10 %.
   - The above conclusion would hold even at fantastic higher energies such as 10^{23} eV, because there is no source of high energy photons; not only π^0 but η mesons cannot decay at such high energies.
2. Photon primary showers.
   - For primary energy $< 2 \times 10^{19}$ eV, only the LPM effect works, and the effect in the shower size at sea level is 30$\sim$40 %
   - For primary energy $4 \sim 9 \times 10^{19}$ eV, the LPM and geomagnetic effect compete. Showers with small $B \sin\theta$ (coming from the south) and with large one (coming from the north) have quite different shower sizes (factor 2$\sim$4. north-south asymmetry). Showers coming from the north have very small fluctuation and larger sizes, contrasting to those from the south.
   - One would need an elaborated method for *correct* energy estimation for such showers, especially, if the observation is an Akeno type one layer sampling calorimeter.
   - If one want to see the north-south asymmetry, there is an optimum height depending on the energy and the zenith angle of incoming photons. A higher land would give clear signals in general.
   - For primary energy $\gtrsim 1.5 \times 10^{20}$ eV, the geomagnetic effect dominates, and the fluctuation of shower development becomes very small. The north-south asymmetry is diminished.
3. We need more simulations if we assume a given event is initiated by a photon. Also we must input experimental site information in the simulation. We must examine the energy estimation method whether it gives correct value.
4. One must estimate the percentage of photon primaries if the north-south asymmetry is to be observable.
5. Every observational data of high energy showers should be provided with the arrival direction expressed in the zenith and azimuthal angles.
6. If photons are abundant at $> 10^{19}$ eV, the primary energy of showers with energy $< 3 \times 10^{19}$ eV is tend to be underestimated because of the LPM effect, while the higher energy ones are tend to be overestimated due to the geomagnetic effect except for those showers that come along the magnetic field line. Therefore, we may have a dip in the primary energy spectrum at around 3×10^{19} eV. It is interesting to note that the present Akeno data seems to have such a dip at $\sim 4 \times 10^{19}$ eV, although we will need 10 times more statistics to confirm such a dip.

Acknowledgment

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